Complex Basic Operations			
Operation	**Regular form**	**Polar to Cartesian form**	**Exponential form**
z	$a+ib$	$r(\cos\theta+i\sin\theta)$	$re^{i\theta}$
z_1+z_2	$(a+c)+\mathrm{i}(\mathrm{b}+\mathrm{d})$	$\sqrt{(a+c)^2+(b+d)^2}\angle\tan^{-1}\left(\frac{b+d}{a+c}\right)$	$r_1e^{i\theta_1}+r_2e^{i\theta_2}$
z_1-z_2	$(\mathrm{a}-\mathrm{c})+i(b-d)$	$\sqrt{(a-c)^2+(b-d)^2}\angle\tan^{-1}\left(\frac{b-d}{a-c}\right)$	$r_1e^{i\theta_1}-r_2e^{i\theta_2}$
z_1z_2	$(ac-bd)+i(ad+bc)$	$r_1r_2[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)]$	$r_1r_2e^{i(\theta_1+\theta_2)}$
$\frac{z_1}{z_2}$	$\frac{(ac+bd)+i(bc-ad)}{c^2+d^2}$	$\frac{r_1}{r_2}[cos(\theta_1-\theta_2)+i\,sin(\theta_1-\theta_2)]$	$\frac{r_1}{r_2}e^{i(\theta_1-\theta_2)}$
$\frac{1}{z}$	$\frac{a}{a^2+b^2}-i\frac{b}{a^2+b^2}$	$\frac{1}{r}(\cos\theta-i\sin\theta)$	$\frac{1}{r}e^{-i\theta}$
z^2	$(a^2-b^2)+i2ab$	$r^2(\cos 2\theta+i\sin 2\theta)$	$r^2e^{i2\theta}$
$\sqrt{z}$	$\frac{1}{\sqrt{2}}\left(\sqrt{r+a}+i\sqrt{r-a}\right)$	$\sqrt{r}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)$	$\sqrt{r}e^{i\frac{\theta}{2}}$
Z^n	$(a+ib)^n$	$r^n(\cos n\theta+i\sin n\theta)$	$r^ne^{in(\theta+2m\pi)}$
$\sqrt[n]{z}$	$\sqrt[n]{(a+ib)}$	$\sqrt[n]{r}\left(\cos\frac{\theta+2k\pi}{n}+i\sin\frac{\theta+2k\pi}{n}\right)$	$r^{\frac{1}{n}}e^{i\left(\frac{\theta+2k\pi}{n}\right)}$
${z_1}^{Z_2}$	$(a+ib)^{(c+id)}=(a^2+b^2)^{\frac{(c+id)}{2}}e^{i(c+id)\theta}$ $\mathrm{r^ce^{-d\theta}}[\cos(d\ln r+c\theta+2ck\pi)+i\sin(d\ln r+c\theta+2ck\pi)]$		$e^{Z_2\ln(Z_1)}$
$\ln z$	$\ln\left(re^{i\theta}\right)=\ln\left[re^{i(\theta+2n\pi)}\right]=\ln r+i(\theta+2n\pi)$ $\quad z\neq 0$		
$\log_{z_2}z_1$	$\frac{\ln z_1}{\ln z_2}=\frac{\ln(a+ib)}{\ln(c+id)}$		
x^z	$x^a[\cos(b\,ln\,x)+i\sin(b\,ln\,x)]$		$e^{z\ln x}=x^ae^{i(b\ln x)}$
e^z	$e^a(\cos b+i\sin b)$	$e^{z+i2\pi n}$	e^ae^{ib}
$\bar{z}$ *conjugate*	$a-ib$	$r(\cos\theta-i\sin\theta)$	$re^{-i\theta}$

$z_1=a+ib$ $\quad z_2=c+id$ $\quad \arg(z)=\theta=\tan^{-1}\left(\frac{b}{a}\right)+2n\pi$ $\quad r=\sqrt{a^2+b^2}$ $\quad k=0,1\ldots.n-1$

$m,n=0,1,2\ldots any\ integer$

Complex Trigonometric Functions			
$\sin z$	$\sin(a)\cosh(b) + i\cos(a)\sinh(b)$	$\frac{e^{iz} - e^{-iz}}{i2}$	$-i\sinh(iz)$
$\cos z$	$\cos(a)\cosh(b) - i\sin(a)\sinh(b)$	$\frac{e^{iz} + e^{-iz}}{2}$	$\cosh(iz)$
$\tan z$	$\frac{\sin z}{\cos z}$	$\frac{(e^{iz} - e^{-iz})}{i(e^{iz} + e^{-iz})}$	$-i\tanh(iz)$
$\cot z$	$\frac{\cos z}{\sin z} = \frac{1}{\tan z}$	$\frac{i(e^{iz} + e^{-iz})}{e^{iz} - e^{-iz}}$	$i\coth(iz)$
$\sec z$	$\frac{1}{\cos z}$	$\frac{2}{e^{iz} + e^{-iz}}$	$\operatorname{sech}(iz)$
$\csc z$	$\frac{1}{\sin z}$	$\frac{i2}{e^{iz} - e^{-iz}}$	$i\operatorname{csch}(iz)$

Complex Hyperbolic Functions			
$\sinh z$	$\sinh(a)\cos(b) + i\cosh(a)\sin(b)$	$\frac{e^{z} - e^{-z}}{2}$	$-i\sin(iz)$
$\cosh z$	$\cosh(a)\cos(b) + i\ \sinh(a)\sin(b)$	$\frac{e^{z} + e^{-z}}{2}$	$\cos(iz)$
$\tanh z$	$\frac{\sinh z}{\cosh z}$	$\frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$	$-i\tan(iz)$
$\coth z$	$\frac{\cosh z}{\sinh z} = \frac{1}{\tanh z}$	$\frac{e^{z} + e^{-z}}{e^{z} - e^{-z}}$	$i\cot(iz)$
$\operatorname{sech} z$	$\frac{1}{\cosh z}$	$\frac{2}{e^{z} + e^{-z}}$	$\sec(iz)$
$\operatorname{csch} z$	$\frac{1}{\sinh z}$	$\frac{2}{e^{z} - e^{-z}}$	$i\csc(iz)$

Complex Inverse Trigonometric Functions				
$\arcsin z$	$-i\ln\left(iz \pm \sqrt{1 - z^2}\right)$	$\frac{\pi}{2} - \cos^{-1} z$	$\csc^{-1}\left(\frac{1}{z}\right)$	$-i\sinh^{-1}(iz)$
$\arccos z$	$-i\ln\left(z \pm i\sqrt{1 - z^2}\right)$	$\frac{\pi}{2} - \sin^{-1} z$	$\sec^{-1}\left(\frac{1}{z}\right)$	$\pm i\cosh^{-1}(z)$
$\arctan z$	$\frac{i}{2}\ln\left(\frac{i + z}{i - z}\right) = \frac{i}{2}\ln\left(\frac{1 - iz}{1 + iz}\right)$	$\frac{\pi}{2} - \cot^{-1} z$	$\cot^{-1}\left(\frac{1}{z}\right)$	$-i\tanh^{-1}(iz)$
$\operatorname{arccot} z$	$\frac{1}{2i}\ln\left(\frac{z + i}{z - i}\right)$	$\frac{\pi}{2} - \tan^{-1} z$	$\tan^{-1}\left(\frac{1}{z}\right)$	$i\coth^{-1}(iz)$
$\operatorname{arcsec} z$	$\frac{1}{i}\ln\left(\frac{1 + \sqrt{1 - z^2}}{z}\right)$	$\frac{\pi}{2} - \csc^{-1} z$	$\cos^{-1}\left(\frac{1}{z}\right)$	$\pm i\operatorname{sech}^{-1}(z)$
$\operatorname{arccsc} z$	$\frac{1}{i}\ln\left(\frac{i + \sqrt{z^2 - 1}}{z}\right)$	$\frac{\pi}{2} - \sec^{-1} z$	$\sin^{-1}\left(\frac{1}{z}\right)$	$i\operatorname{csch}^{-1}(iz)$

Complex Inverse Hyperbolic Functions			
$\text{arc}\sinh z$	$\ln\left(z \pm \sqrt{z^2+1}\right)$	$\operatorname{csch}^{-1}\left(\frac{1}{z}\right)$	$-i\sin^{-1}(iz)$
$\text{arc}\cosh z$	$\ln\left(z \pm \sqrt{z^2-1}\right) = \ln\left(z+\sqrt{z+1}\sqrt{z-1}\right)$	$\operatorname{sech}^{-1}\left(\frac{1}{z}\right)$	$\pm i\cos^{-1}(z)$
$\text{arc}\tanh z$	$\frac{1}{2}\ln\left(\frac{1+z}{1-z}\right) = \frac{1}{2}[\ln(1+z)-\ln(1-z)]$	$\coth^{-1}\left(\frac{1}{z}\right)$	$-i\tan^{-1}(iz)$
$\text{arc}\coth z$	$\frac{1}{2}\ln\left(\frac{z+1}{z-1}\right) = \frac{1}{2}\left[\ln\left(1+\frac{1}{z}\right)-\ln\left(1-\frac{1}{z}\right)\right]$	$\tanh^{-1}\left(\frac{1}{z}\right)$	$i\cot^{-1}(iz)$
$\text{arc}\operatorname{sech} z$	$\ln\left(\frac{1\pm\sqrt{1-z^2}}{z}\right)$	$\cosh^{-1}\left(\frac{1}{z}\right)$	$\pm i\sec^{-1}(z)$
$\text{arc}\operatorname{csch} z$	$\ln\left(\frac{1\pm\sqrt{1+z^2}}{z}\right)$	$\sinh^{-1}\left(\frac{1}{z}\right)$	$i\csc^{-1}(iz)$

Trigonometric Relations	
$\sin(z_1 \pm z_2)$	$\sin z_1 \cos z_2 \pm \cos z_1 \sin z_2$
$\cos(z_1 \pm z_2)$	$\cos z_1 \cos z_2 \mp \sin z_1 \sin z_2$
$\tan(z_1 \pm z_2)$	$\frac{\tan z_1 \pm \tan z_2}{1 \mp \tan z_1 \tan z_2}$

Complex Trigonometric and Hyperbolic Function Relations			
$\sin z$	$\sin(z+2\pi n)$	$-\sin(-z)$	$\cos\left(\frac{\pi}{2}-z\right) = -\cos\left(\frac{\pi}{2}+z\right)$
$\cos z$	$\cos(z+2\pi n)$	$\cos(-z)$	$\sin\left(\frac{\pi}{2}+z\right) = \sin\left(\frac{\pi}{2}-z\right)$
$\tan z$	$\tan(z+\pi n)$	$-\tan(-z)$	$\cot\left(\frac{\pi}{2}-z\right) = -\cot\left(\frac{\pi}{2}+z\right)$
$\cot z$	$\cot(z+\pi n)$	$-\cot(-z)$	$\tan\left(\frac{\pi}{2}-z\right) = -\tan\left(\frac{\pi}{2}+z\right)$
$\csc z$	$\csc(z+2\pi n)$	$-\csc(-z)$	$\sec\left(\frac{\pi}{2}-z\right) = -\sec\left(\frac{\pi}{2}+z\right)$
$\sec z$	$\sec(z+2\pi n)$	$\sec(-z)$	$\csc\left(\frac{\pi}{2}-z\right) = \csc\left(\frac{\pi}{2}+z\right)$

$\sinh(-z) = -\sinh z$
$\cosh(-z) = \cosh z$
$\tanh(-z) = -\tanh z$

$\sin iz = i \sinh z$
$\cos iz = \cosh z$
$\tan iz = i \tanh z$

$\sinh iz = i \sin z$
$\cosh iz = \cos z$
$\tanh iz = i \tan z$

$\sin^2 z + \cos^2 z = 1$
$1 + \tan^2 z = \sec^2 z$
$1 + \cot^2 z = \csc^2 z$

$cosh^2 z - sinh^2 z = 1$
$1 - tanh^2 z = sech^2 z$
$\coth^2 z - 1 = \operatorname{csch}^{2-1} z$

$\sin 2z = 2 \sin z \cos z$
$\cos 2z = \cos^2 z - \sin^2 z$
$\tan 2z = \dfrac{2 \tan z}{1 - \tan^2 z}$

$\sin\left(\frac{z}{2}\right) = \sqrt{\frac{1 - \cos z}{2}}$	$\sinh\left(\frac{z}{2}\right) = \sqrt{\frac{\cosh(z) - 1}{2}}$
$\cos\left(\frac{z}{2}\right) = \sqrt{\frac{1 + \cos z}{2}}$	$\cosh\left(\frac{z}{2}\right) = \sqrt{\frac{1 + \cosh z}{2}}$
$\tan\left(\frac{z}{2}\right) = \frac{1 - \cos z}{\sin z} = \frac{\sin z}{1 + \cos z}$	$\tanh\left(\frac{z}{2}\right) = \frac{1 - \cosh z}{\sinh z}$

$$\cos(2z) + i \sin(2z) = (\cos^2 z - \sin^2 z) + i(2 \sin z \cos z) = e^{i2z}$$

Powers	
$i^0 = 1$	$i^0 = 1$
$i^1 = i$	$i^{-1} = -i$
$i^2 = -1$	$i^{-2} = -1$
$i^3 = -i$	$i^{-3} = i$
$i^4 = 1$	$i^{-4} = 1$
$i^5 = i$	$i^{-5} = -i$
$i^6 = -1$	$i^{-6} = -1$
$i^7 = -i$	$i^{-7} = i$
$i^8 = 1$	$i^{-8} = 1$
$i^9 = i$	$i^{-9} = -i$
$i^{10} = -1$	$i^{-10} = -1$
General	
$i^{4n} = e^{i(2n)\pi} = 1$	$i^{-4n} = -i$
$i^{4n+1} = e^{i\left(2n+\frac{1}{2}\right)\pi} = i$	$i^{-(4n+1)} = -1$
$i^{4n+2} = e^{i(2n+1)\pi} = -1$	$i^{-(4n+2)} = i$
$i^{4n+3} = e^{i\left(2n+\frac{3}{2}\right)\pi} = -i$	$i^{-(4n+3)} = 1$

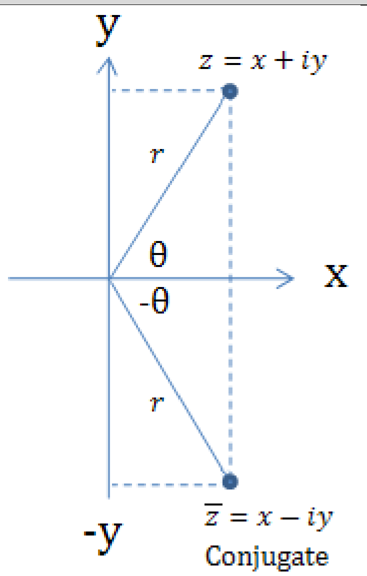
n=0, 1, 2 … (integer number)

Logarithms

$\ln z = \ln\lvert z\rvert + i[Arg\ z + 2k\pi]$ $k = 0, \pm 1, \pm 2 \ldots.$	$z = e^{\ln z}$
$\ln\left(\frac{1}{z}\right) = -\ln z$	$y^x = e^{x \ln y}$
$\ln(z_1 z_2) = \ln z_1 + \ln z_2$	$z^a = e^{a \ln z}$
$\ln\left(\frac{z_1}{z_2}\right) = \ln z_1 - \ln z_2$	$e^{-z} = \frac{1}{e^z}$
$\log_n z = \frac{\log z}{\log n}$	$\boldsymbol{e^{i\pi} = -1}$
$\log_i x = \frac{-i2 \ln x}{\pi}$	$\lvert e^z\rvert = e^x$
$\overline{\log z} = \log(\bar{z})$	$i^i = e^{i \ln i} = e^{-\frac{\pi}{2}+2\pi n}$
$\ln(e^z) = z + i2n\pi$	$e = 2.71828183 \ldots$
$\ln(1) = 0$	$e^{i\theta} = \cos\theta + i\sin\theta$
$\ln(-1) = \ln(e^{i\pi}) = i\pi$	$e^{iz} = \cos z + i \sin z$
$\ln(0) = -infinity$	$e^{-iz} = \cos z - i\sin z$
$\ln(i) = i\left(\frac{\pi}{2} + 2k\pi\right)$	$e^z = \cosh z + \sinh z$
$\ln(-i) = -i\left(\frac{\pi}{2} + 2k\pi\right)$	
$\ln(\infty) = infinity$	$\ln(-a) = \ln a + i\pi$

Conjugate & Modulus

$\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$	$z + \bar{z} = 2a$
$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$	$z - \bar{z} = i2b$
$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2} \quad if\ z_2 \neq 0$	$z\bar{z} = x^2 + y^2 = \lvert z\rvert^2$
$\overline{(\bar{z})} = z$	$\frac{z}{\bar{z}} = e^{i2\theta}$
$\lvert z\rvert = \lvert\bar{z}\rvert = \sqrt{x^2 + y^2}$	$z = \bar{z} \quad only\ if\ z\ is\ real$
$\left\lvert\frac{z}{\bar{z}}\right\rvert = 1$	$\bar{z} = -z \ \ if\ z\ is\ imaginary$
$\lvert z\rvert^2 = z\bar{z} = \bar{z}z$	$\bar{i} = -i$
$z^{-1} = \bar{z}\lvert z\rvert^{-2} \qquad z \neq 0$	$\lvert i\rvert = 1$
$\lvert z_1 + z_2\rvert \leq \lvert z_1\rvert + \lvert z_2\rvert$	$\lvert z\rvert\ modulus\ of\ z$
$\lvert z_1\rvert - \lvert z_2\rvert \leq \lvert z_1 - z_2\rvert$	$\lvert z\rvert = \sqrt{z\bar{z}}$
$\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$	$arg(z) = \tan^{-1}\frac{z - \bar{z}}{z + \bar{z}}$



Conjugate – Polar Form

$$\bar{z} = re^{-i\theta}$$

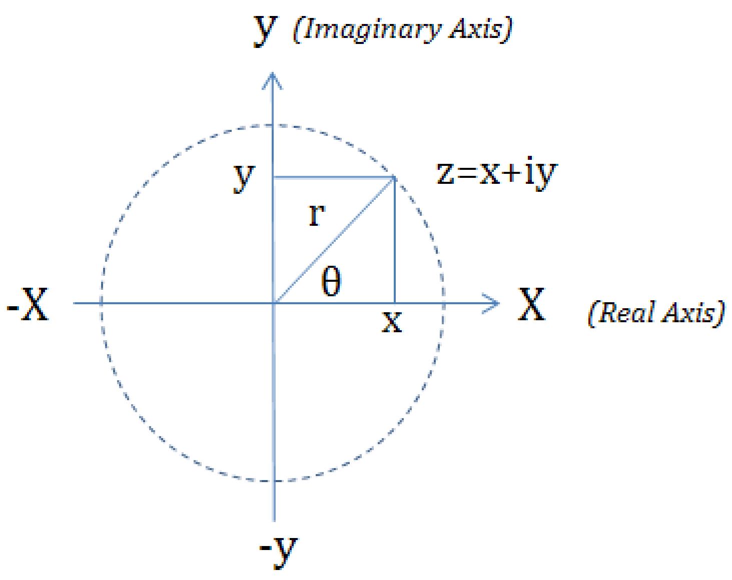
Conjugate – Polar to Cartesian

$$\bar{z} = r(\cos\theta - i\sin\theta)$$

Cartesian Coordinate

A complex number can be viewed as a point or a vector on a two dimensional Cartesian coordinate system, called the complex plane or Argand diagram.

$$z = x + iy$$



Polar Coordinate

The polar coordinate system uses angle and vector length r of a point from the origin of the axe. $z = re^{i\theta}$

r - modulus (absolute value of z)

$$r = \sqrt{x^2 + y^2}$$

θ - phase or the argument of z

$$z = \arg(z) = \tan^{-1}\frac{y}{x}$$

Polar to Cartesian

$$z = r(\cos\theta + i\sin\theta)$$

Complex Derivative

Derivative of a complex function *f(z)* at a point z_0

$$\frac{df}{dz} = f'(z_0) = \lim_{\Delta z \to 0}\frac{f(z_0 + \Delta z) - f(z_0)}{\Delta z}$$

Δz *approch zero in any way possible*

Cauchy Riemann Equations

A necessary condition for a function

$$f(z) = u(x, y) + iv(x, y)$$

To be analytic at a point $z=x+iy$ is that the Cauchy Riemann equations are satisfied at that point.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$(u_x = v_y) \qquad (u_y = -v_x)$$

Cauchy Equation in Polar Form

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \qquad \frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$$

Harmonic Function

Function f(x,y) is harmonic if the first and second partial derivatives are continuous in a given domain and the Laplace's equation is satisfied.

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

$$(f_{xx} + f_{yy} = 0)$$

Harmonic Complex Function

The real and the imaginary parts of a complex function is satisfying the Laplace equation

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0 \qquad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

Cauchy Integral

If *f(z)* is analytic in a closed domain *C* and z_0 is a point interior to C then:

$$f(z_0) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z - z_0}dz$$

Taylor Series

If *f(z)* is infinitely differentiable at a point z_0 then it can be expressed as a power series.

$$f(z_0) = \frac{f'(z_0)}{1!}(z - z_0) + \frac{f''(z_0)}{2!}(z - z_0)^2 + \ldots$$

$$f(z_0) = \sum_{n=0}^{\infty}\frac{f^{(n)}(z_0)}{n!}(z - z_0)^n$$

Oscillation Complex number describing oscillation. $\omega -$ *angular velocity* t *– time* $\varphi -$ *initial phase of starting point*	$e^{j(\omega t+\varphi)} = \cos(\omega t+\varphi) + j\sin(\omega t+\varphi)$

Series expansion of complex functions	
$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots + \frac{z^n}{n!} = 1 + \sum_{n=1}^{\infty} \frac{z^n}{n!}$	$for\ all\ z$
$e^{-z} = 1 - z + \frac{z^2}{2!} - \frac{z^3}{3!} + \ \ldots = \sum_{n=1}^{\infty} (-1)^n \frac{z^n}{n!}$	$for\ all\ z$
$e^{\frac{1}{z}} = 1 + \frac{1}{z} + \frac{1}{2!\ z^2} + \frac{1}{3!\ z^2} + \ \ldots = \sum_{n=0}^{\infty} \frac{1}{n!\, z^n}$	$z \neq 0$
$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \ \ldots = 1 + \sum_{n=1}^{\infty} \frac{(i\theta)^n}{n!}$	
$\ln z = (z-1) - \frac{(z-1)^2}{2} + \frac{(z-1)^3}{3} - \frac{(z-1)^4}{4} + \ \ldots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(z-1)^n}{n}$	
$\ln(1-z) = -z - \frac{z^2}{2} - \frac{z^3}{3} - \frac{z^4}{4} - \ \ldots = -\sum_{n=1}^{\infty} \frac{z^n}{n}$	$for\ \lvert z \rvert < 1$
$\ln(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \ \ldots = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{z^n}{n}$	$for\ \lvert z \rvert \leq 1 \quad z \neq -1$
$\ln\frac{1}{1-z} = z + \frac{z^2}{2} + \frac{z^3}{3} + \frac{z^4}{4} + \ \ldots = \sum_{n=1}^{\infty} \frac{z^n}{n}$	$\lvert z \rvert \leq 1$
$\ln\frac{z}{z-1} = \frac{1}{z} + \frac{1}{2z^2} + \frac{1}{3z^3} + \ \ldots = \sum_{n=1}^{\infty} \frac{1}{nz^n}$	$\lvert z \rvert > 1$
$\frac{1}{1-z} = 1 + z + z^2 + z^3 + \ \ldots = \sum_{n=0}^{\infty} z^n$	$for\ \lvert z \rvert < 1$
$\frac{1}{1+z} = 1 - z + z^2 - z^3 + \cdots = \sum_{n=0}^{\infty} (-1)^n z^n$	$\lvert z \rvert < 1$
$\frac{z}{(1-z)^2} = z + 2z^2 + 3z^3 + 4z^4 + \ \ldots = \sum_{n=1}^{\infty} nz^n$	$for\ \lvert z \rvert < 1$

$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$	$-\infty < z < \infty$
$\sin^{-1} z = z + \left(\frac{1}{2}\right)\frac{z^3}{3} + \left(\frac{1\cdot 3}{2\cdot 4}\right)\frac{z^5}{5} + \left(\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)\frac{z^7}{7}\right) + \ldots\ldots = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2}\frac{z^{2n+1}}{(2n+1)}$	$for\ \|z\| \le 1$
$\sinh z = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \frac{z^7}{7!} + \ldots = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}$	$for\ all\ z$
$\sinh^{-1} z = z - \frac{1}{6}z^3 + \frac{3}{40}z^5 - \frac{5}{112}z^7 + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{(2n)!}{4^n (n!)^2 (2n+1)} z^{2n+1}$	$for\ \|z\| < 1$

$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$	$-\infty < z < \infty$
$\cos^{-1} z = \frac{\pi}{2} - z - \frac{1}{6}z^3 - \frac{3}{40}z^5 - \frac{5}{112}z^7 - \ldots = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2}\frac{z^{2n+1}}{(2n+1)}$	$for\ \|z\| \le 1$
$\cosh z = 1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \frac{z^6}{6!} + \ldots = 1 + \sum_{n=1}^{\infty} \frac{z^{2n}}{(2n)!}$	$for\ all\ z$
$\cosh^{-1} z = \ln(2z) - \frac{1}{4z^2} - \frac{3}{32z^4} - \frac{15}{288z^6} - \ldots$	$for\ \|z\| > 1$

$\tan z = z + \frac{1}{3}z^3 + \frac{2}{15}z^5 + \frac{17}{315}z^7 + \frac{62}{2835}z^9 \ldots = \sum_{n=1}^{\infty} \frac{B_{2n}(1-4^n)}{(2n)!} z^{2n-1}$	$\|z\| < \frac{\pi}{2}$
$\tan^{-1} z = z - \frac{z^3}{3} + \frac{z^5}{5} - \frac{z^7}{7} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{2n+1}$	$\|z\| \le 1 \quad z \ne i, \quad z \ne -i$
$\tanh z = z - \frac{1}{3}z^3 + \frac{2}{15}z^5 - \frac{17}{315}z^7 + \ldots = \sum_{n=1}^{\infty} \frac{B_{2n}4^n(4^n-1)}{(2n)!} z^{2n-1}$	$for\ \|z\| < \frac{\pi}{2}$
$\tanh^{-1} z = z + \frac{z^3}{3} + \frac{z^5}{5} + \frac{z^7}{7} + \ldots = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{2n+1}$	$for\ \|z\| < 1$

$$\cot z = \frac{1}{z} - \frac{1}{3}z - \frac{1}{45}z^3 - \frac{2}{945}z^5 - \frac{1}{4725}z^7 - \ \dots = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} B_{2n} z^{2n-1}}{(2n)!} \qquad for\ \ 0 < |z| < \pi$$

$$\cot^{-1} z = \frac{\pi}{2} - \tan^{-1} z = \frac{\pi}{2} - \left(z - \frac{z^3}{3} + \frac{z^5}{5} - \frac{z^7}{7} + \ \dots.\right) = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{2n+1} \qquad |z| \le 1 \quad z \ne \pm i$$

$$\coth z = \frac{1}{z} + \frac{1}{3}z - \frac{1}{45}z^3 + \frac{2}{945}z^5 - \frac{1}{4725}z^7 + \ \dots = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{2^{2n} B_{2n} z^{2n-1}}{(2n)!} \qquad 0 < |z| < \pi$$

$$\coth^{-1} z = \tanh^{-1}\left(\frac{1}{z}\right)$$

$$\csc z = \frac{1}{z} + \frac{1}{6}z + \frac{7}{360}z^3 + \frac{31}{15120}z^5 + \ \dots = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2(2^{2n-1} - 1) B_{2n} z^{2n-1}}{(2n)!} \qquad for\ \ 0 < |z| < \pi$$

$$\csc^{-1} z = \sin^{-1}\left(\frac{1}{z}\right) = \frac{1}{z} + \left(\frac{1}{2}\right)\frac{1}{3z^3} + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)\frac{1}{5z^5} + \ \dots = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2} \frac{1}{(2n+1) z^{(2n+1)}}$$

$$\operatorname{csch} z = \frac{1}{z} - \frac{1}{6}z + \frac{7}{360}z^3 - \frac{31}{15120}z^5 + \ \dots = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{2\left(1 - 2^{(2n-1)}\right)}{(2n)!} B_{2n} z^{(2n-1)} \qquad 0 < |z| < \pi$$

$$\operatorname{csch}^{-1} z = \ln 2 - \ln z + \frac{1}{4}z^2 - \frac{3}{32}z^4 + \frac{5}{96}z^6 - \ \dots$$

$$\sec z = 1 - \frac{1}{2}z^2 + \frac{5}{24}z^4 - \frac{61}{720}z^6 + \frac{277}{8064}z^8 - \ \dots = \sum_{n=0}^{\infty} (-1)^n \frac{E_{2n}}{(2n)!} z^{2n} \qquad for\ \ |z| < \frac{\pi}{2}$$

$$\sec^{-1} z = \frac{\pi}{2} - \left(\frac{1}{z} + \left(\frac{1}{2}\right)\frac{1}{3z^3} + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)\frac{1}{5z^5} + \cdots\right) = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2} \frac{1}{(2n+1) z^{(2n+1)}} \qquad |z| \ge 1$$

$$\operatorname{sech} z = 1 - \frac{1}{2}z^2 + \frac{5}{24}z^4 - \frac{61}{720}z^6 + \frac{277}{8064}z^8 + \ \dots = \sum_{n=0}^{\infty} \frac{E_{2n} z^{2n}}{(2n)!} \qquad |z| < \frac{\pi}{2}$$

$$\operatorname{sech}^{-1} z = \ln 2 - \ln z - \frac{1}{4}z^2 - \frac{3}{32}z^4 \ \dots.$$

B_n *nth Bernoulli number*: $B_0 = 1 \quad B_1 = -\frac{1}{2} \quad B_2 = \frac{1}{6} \quad B_4 = -\frac{1}{30} \quad B_6 = \frac{1}{42} \quad B_8 = -\frac{1}{30}$

$B_{10} = \frac{5}{66} \quad B_{12} = -\frac{691}{2{,}730} \quad B_{14} = \frac{7}{6} \quad B_{16} = -\frac{3{,}617}{510} \quad B_{18} = \frac{43{,}867}{798}$

E_n *nth Euler number*: $E_0 = 1 \quad E_2 = -1 \quad E_4 = 5 \quad E_6 = -61 \quad E_8 = 1{,}385 \quad E_{10} = -50{,}521$
$E_{12} = 2{,}702{,}765 \quad E_{14} = -199{,}360{,}981 \quad E_{16} = 19{,}391{,}512{,}145$

Derivatives of inverse trigonometric and hyperbolic functions	
$\frac{d}{dz}\sin^{-1}(z) = \frac{1}{\sqrt{1-z^2}}$	$\frac{d}{dz}\sinh^{-1}(z) = \frac{1}{\sqrt{1+z^2}}$
$\frac{d}{dz}\cos^{-1}(z) = -\frac{1}{\sqrt{1-z^2}}$	$\frac{d}{dz}\cosh^{-1}(z) = \frac{1}{\sqrt{z+1}\sqrt{z-1}}$
$\frac{d}{dz}\tan^{-1}(z) = \frac{1}{1+z^2}$	$\frac{d}{dz}\tanh^{-1}(z) = \frac{1}{1-z^2}$
$\frac{d}{dz}\cot^{-1}(z) = -\frac{1}{1+z^2}$	$\frac{d}{dz}\coth^{-1}(z) = \frac{1}{1-z^2}$
$\frac{d}{dz}\csc^{-1}(z) = -\frac{1}{z\sqrt{z^2-1}}$	$\frac{d}{dz}\operatorname{csch}^{-1}(z) = -\frac{1}{z^2\sqrt{1+\frac{1}{z^2}}}$
$\frac{d}{dz}\sec^{-1}(z) = \frac{1}{z\sqrt{z^2-1}}$	$\frac{d}{dz}\operatorname{sech}^{-1}(z) = -\frac{1}{z(z+1)\sqrt{\frac{1-z}{1+z}}}$

Derivatives of trigonometric and hyperbolic functions	
$\frac{d}{dz}\sin(z) = \cos(z)$	$\frac{d}{dz}\sinh(z) = \cosh(z)$
$\frac{d}{dz}\cos(z) = -\sin(z)$	$\frac{d}{dz}\cosh(z) = \sinh(z)$
$\frac{d}{dz}\tan(z) = \sec^2(z)$	$\frac{d}{dz}\tanh(z) = 1-\tanh^2(z) = \operatorname{sech}^2(z)$
$\frac{d}{dz}\cot(z) = -\csc^2(z)$	$\frac{d}{dz}\coth(z) = -\operatorname{csch}^2(z)$
$\frac{d}{dz}\csc(z) = -\csc(z)\cot(z)$	$\frac{d}{dz}\operatorname{csch}(z) = -\operatorname{csch}(z)\coth(z)$
$\frac{d}{dz}\sec(z) = \sec(z)\tan(z)$	$\frac{d}{dz}\operatorname{sech}(z) = -\operatorname{sech}(z)\tanh(z)$

Derivatives of common complex functions	
$\frac{d}{dz}z^a = az^{a-1}$	$\frac{d}{dz}z^z = z^z(1+\ln z)$
$\frac{d}{dz}a^z = a^z\ln(a)$	$\frac{d}{dz}\ln z = \frac{1}{z}$
$\frac{d}{dz}e^z = e^z$	$\frac{d}{dz}\log_a z = \frac{1}{z\ln a}$

Software documentation

One source of confusion when dealing with complex numbers are the multiple solutions to several complex functions calculations, this software manages multiple solutions as follows:

Function	AmBrSoft – Complex Calculator
$\arg(z) = \theta$	The software enables the use of two presentation of argument angles: **Present of negative (regular) angles** $-\pi < \theta \leq \pi$ This is the default setup for calculations and corresponds to most definitions of complex calculations. **Present positive angles** $0 < \theta < 2\pi$ In this way all angles will appear as positive values counterclockwise from the real x axis.
$\ln(z) = \ln r + i(\theta + 2k\pi)$	θ is restricted to a single value (k=0) in the range: For negative angles (default) $-\pi < \theta < \pi$ For positive angles $0 < \theta < 2\pi$
e^z	Has a single value
$\sin z,\ \cos z,\ \tan z,\ \cot z,\ \sec z,\ \csc z$	Has a single value
$\sin^{-1} z, \cos^{-1} z, \sec^{-1} z, \csc z$	Has 2 solutions in the range
$\tan^{-1} z, \cot^{-1} z$	Solution is single value
z^n	Has a single value if n is integer
$\sqrt[n]{z}$	Solution has n values and the angle differs by $\frac{2\pi}{n}$ radians User can browse between different solutions by pressing The button.
Calculations accuracy	All calculations are performed up to 16 digits.